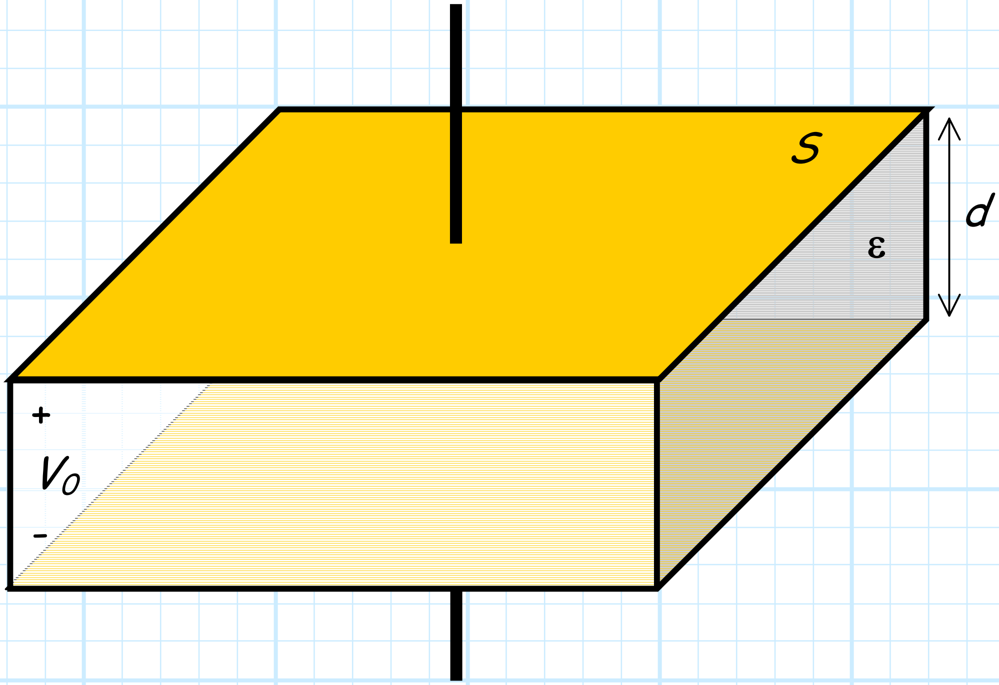
The Parallel Plate Capacitor

Consider the geometry of a **parallel plate capacitor**:



Where:

V_0 = the **potential difference** between the plates
S = **surface area** of each conducting plate
d = **distance** between plates
ε = **permittivity** of the dielectric between the plates

Recall that we determined the fields and surface charge density of an **infinite** pair of parallel plates. We can use those results to approximate the fields and charge densities of this **finite** structure, where the **area** of each plate is S.

For example, we determined that the **surface charge density** on the upper plate is:

$$\rho_{s+}(\bar{r}) = \frac{\varepsilon V_0}{d}$$

The **total charge** on the upper plate is therefore:

$$\begin{aligned} Q &= \iint_{S_+} \rho_{s+}(\bar{r})\, ds \\ &= \iint_{S_+} \frac{\varepsilon V_0}{d}\, ds \\ &= \frac{\varepsilon V_0}{d} \iint_{S_+} ds \\ &= \frac{\varepsilon V_0 S}{d} \end{aligned}$$

The **capacitance** of this structure is therefore:

$$C = \frac{Q}{V} = \left(\frac{\varepsilon V_0 S}{d}\right)\left(\frac{1}{V_0}\right) = \frac{\varepsilon S}{d} \quad [\text{Farads}]$$

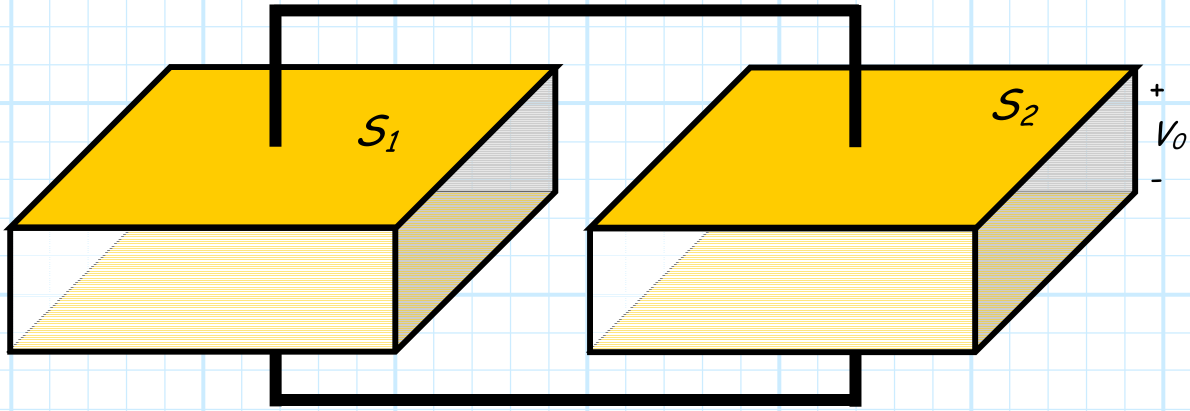
Note therefore, that we can **increase** the capacitance of a parallel plate capacitor by:

1) **Increasing** surface area *S*.

2) Decreasing separation distance *d*.

3) **Increasing** the dielectric permittivity ε.

Consider now the structure:



Note the **two** upper plates form **one** conducting structure, and the **two** bottom plates form **another**.

Q: *What is the **capacitance** between these two conducting structures?*

A: The potential difference between them is V_0. The **total charge** on one conducting structure is simply the **sum** of the charges on **each plate**:

$$
\begin{aligned}
Q = Q_1 + Q_2 &= \frac{\varepsilon V_0 S_1}{d} + \frac{\varepsilon V_0 S_2}{d} \\
&= \frac{\varepsilon V_0 \left(S_1 + S_2\right)}{d}
\end{aligned}
$$

Therefore, the **capacitance** of this structure is:

$$
\begin{aligned}
C = \frac{Q}{V} &= \left(\frac{\varepsilon V_0 \left(S_1 + S_2\right)}{d}\right)\left(\frac{1}{V_0}\right) \\
&= \frac{\varepsilon \left(S_1 + S_2\right)}{d} \\
&= \frac{\varepsilon S_1}{d} + \frac{\varepsilon S_2}{d} \\
&= C_1 + C_2
\end{aligned}
$$

But **you** knew this! The total capacitance of two capacitors in **parallel** is equal to the **sum** of **each** capacitance.